1 Chapter 1: Matrices and Systems of Equations

1.1 Systems of Linear Equations

1. systems of linear equations and their geometric interpretation as the intersection of lines
2. inconsistent system: system has no solutions
3. consistent system: at least one solution
4. equivalent systems: if they have the same set of solutions
5. diagonal entries: the numbers on the diagonal of a matrix $a_{11}.a_{22},\ldots,a_{n,n}$
6. strict triangular form of a system: the diagonal entries are nonzero, and the entries below the diagonal are all zero
7. coefficient matrix (or simply the matrix)of a system = the coefficients of the variables in system of equations
8. augmented matrix = the coefficient matrix with the additional column of the entries on the right hand side of the system of equations.
9. elementary row operations = matrix operations that are used to solve the original system of equations. They are: Interchanging two rows, multiplying a row by a nonzero constant, and replacing a row by its sum with a multiple of another row.
10. pivotal row
11. Systems of linear equations can be solve by:

Method 1 back substitution p.6

Method 2 using the elementary row operations on the augmented matrix to reduce it to a strict triangular form p.$8-9$. The elementary row operations to not alter the solution of the system. This method fails if the pivot ends up being 0 at some point, producing an inconsistent system of equations. See new methods in next section.

1.2 Row Echelon Form

1. lead variables
2. free variables
3. row echelon form of a matrix p.15. What's the difference between the strict triangular form and the row echelon form?
4. Gaussian elimination –Method 3 for solving a system of equations using row operations to put a matrix into row echelon form.
5. reduced row echelon form of a matrix p.15. What's the difference between the strict triangular form, the row echelon form, and the reduced row echelon form?
6. Gauss-Jordan reduction –Method 4 for solving for **undetermined** systems of equations using row operations to put a matrix into reduced row echelon form.
7. undetermined systems: fewer equations than unknowns
8. overdetermined systems: more equations than unknowns
9. homogeneous systems = systems whose right hand side is zero. Homogeneous systems are always consistent (i.e. they will always have solutions):

   (*a*) If there are the same number of variables as equations, the system has a unique solution: $x_1 = 0, x_2 = 0, \ldots, x_n = 0$.

   (*b*) If there are more variables than equations, then there are infinitely many solutions because of the free variables.

1.3 Matrix Algebra

1. matrix notation: $A = (a_{i,j})$ or $B = (b_{i,j})$
2. vectors = matrices with only one row (row-vector), or only one column (column-vector), and they could be equal: $\begin{pmatrix} 2 \\ 3 \end{pmatrix} = (2, 3)$
3. $\mathbf{R^n}$ is the euclidean $\mathbf{n}-$space whose elements are column vectors (or simply **vectors**)
4. ith row of a matrix is $a(i, :)$, and jth column of a matrix is $a(:, j)$,
5. $A = B$ if $a_{i,j} = b_{i,j}$ for each i and j
6. $\alpha A, A + B, A - B$ versus the product of matrices AB (page 38)
7. linear combination of vectors
8. algebraic rules p 41
9. $AB \neq BA$, i.e. multiplication of matrices is not commutative
10. identity matrix I $= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
11. nonsingular matrix (or invertible matrix): if the matrix has an inverse
12. singular matrix: if it is not invertible
13. transpose of a matrix A^T (swap the columns with the rows) and the algebraic rules for the transposes
14. symmetric matrices: if $A^T = A$

1.4 Elementary Matrices

1. Elementary matrices = matrices obtained by performing one elementary row operation on I. There are three types:

(Type I) interchanging two rows: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

(Type II) constant multiple of a row: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(Type III) adding a multiple of a row to another row: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$

2. E is an elementary matrix $\Rightarrow$ E has an inverse E^{-1} which is also an elementary matrix
3. B is row equivalent to A if multiplying A by a series of elementary matrices we get B
4. The following are equivalent:
   - (a) A is nonsingular
   - (b) $A\mathbf{x} = \mathbf{0}$ has only one solution: $\mathbf{x} = \mathbf{0}$
   - (c) A is row equivalent to I.
5. $A\mathbf{x} = \mathbf{0}$ if A is nonsingular
6. finding A^{-1} using the elementary row operations (page 66).
7. diagonal matrix: if the entries of the diagonal are the only possible nonzero entries of the matrix
8. upper triangular matrix: if the entries below the diagonal are zero (some of the other entries could be zero as well)
9. lower triangular matrix: if the entries above the diagonal are zero (some of the other entries could be zero as well)
10. triangular matrix: if it is either upper or lower triangular matrix (recall strict triangular matrix).
11. skip Triangular Factorization.